Real Numbers

Theorems

1. Let p be a prime number. If p divides a^2, then p divides a, where a is positive integer

2. If p is prime, $\sqrt{p}$ is irrational
$\sqrt{2}, \sqrt{3}, \sqrt{5}, \sqrt{7}$ etc are irrational

3. Let x be a rational number whose decimal expansion terminates. Then x can be expressed in $\frac{p}{q}$ form, where p and q are coprime, the prime factorisation of q is of the form $2^n 5^m$ where n, m are non-negative integers.

4. Let $x = \frac{p}{q}$ be a rational number such that the prime factorisation of q is of the form $2^n 5^m$, where n, m are non-negative integers. Then, x has a decimal expansion which terminates.

5. Let $x = \frac{p}{q}$ be a rational number, such that the prime factorisation of q is not of the form of $2^n 5^m$, where n, m are non-negative integers. Then, x has a decimal expansion which is non-terminating repeating

Prime Factorisation Method

For any two positive integers,
HCF $(a, b) \times$ LCM $(a, b) = a \times b$
For example
$a = pq^2$
$b = p^3q$
HCF $= pq$
LCM $= p^3q^2$

Fundamental Theorem of Arithmetic

Every composite number can be expressed as a product of primes, and this factorisation is unique, apart from the order in which the prime factors occur.
Composite Number $x = p_1 \times p_2 \times p_3 \times \ldots \times p_n$, where $p_1, p_2, \ldots, p_n$ are prime numbers